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OUTSTANDING PEDAGOGICAL PRINCIPLES NOW FUNCTIONING IN HIGH-SCHOOL MATHEMATICS.

Are teachers of mathematics in high schools concerning themselves with ways and means of rendering more efficient professional service? It is the writer's purpose to adduce some facts that may serve partially to answer this question.

In the first place it may be worthy of note that teachers of high-school mathematics are much less given to reliance upon the teaching tactics of their favorite collegiate model than they were a few years since. They do not now listen in silent awe to collegiate preachments on the *what* and *how* of high-school mathematics to any such extent as was recently the practice. There has recently come into existence among high-school teachers a real faith that more psychological teaching is better teaching. In seeking an answer to the question asked above we may then set aside the imitation of collegiate favorites and take up the question of accepted pedagogical and psychological principles of today.

One may even see in the multiplicity of active organizations of teachers of high-school mathematics, in the numerous significant reports of recent date of commissions, councils, committees and bureaus all pointing out the urgent need and sketching the desired trend of improvement in high-school mathematics, in numerous recent textual attempts at reform, etc., that high-school teachers of mathematics are alive to the need of greater efficiency of teaching and are concerning themselves actively to bring it about.

The question however that always remains in one's thought is to what extent are the findings of these reports being accepted and followed even by those who aid in formulating and

advocating them. Is classroom practice reshaping itself in the light of these studies and their recommendations? Let us remember that teaching acts are born of ideas that have become teaching *ideals.* Sound attempts at improvement do not forerun, but follow, ideals of worthier achievement. The teacher will persist in seeking improvement only so long as his ideals of worthier professional performance impel him to it. Hence the question here mooted: What are some of the outstanding pedagogical principles that have recently become acceptable to high-school teachers of mathematics? These principles will function, so to speak, as fulcra for our lever to pry into the present status of reformative practices. Later we may diagnose tendencies, for accepted and applied principles must generate tendencies, but it is doubtful whether anything yet exists of sufficient strength or stretch to warrant the name of tendency.

What psychological or pedagogical principles are accepted in mathematical high school practice may be gauged with some reliability from widely used texts, from reports and monographs of associations and bureaus and from observation of actual classroom practice. While the writer has tested some of the principles stated below, by comparison with these sources, he has sought to derive them largely by personal contacts with some fifty or more siginificant high-school teachers. Accepted principles may then mean either (1) merely assented to intellectually but not definitely used, or (2) accepted and applied in practice.

My first principle is rather a complicated one, but as it is a very consequential one, it may well be set forth as compartly as seems feasible.

Pedagogical and Psychological Principles.

I. *Efficient teaching of high-school mathematics derives its efficiency mainly from the following five fundamentals of teaching technics:*

(*a*) A clear and compelling conception of the master aims and purposes to be striven for in each year of the high school,

(*b*) The ability to judge, to choose and to employ matter and method that will yield an appreciable measure of these aims and purposes with a majority of the pupils,

(*c*) The technical skill to determine objectively from time to time the degree of progress being made toward these aims and purposes,

(*d*) The generalship so to alter teaching strategies in the light of the findings, as to attain the goals of the year with the majority of the pupils,

(*e*) The tact and skill to attain the destined ends without impairing the native taste and enthusiasm for mathematical learning.

It is now difficult, indeed, to find a significant teacher of high-school mathematics who does not concede the psychological justifiability of these five fundamentals. The teacher may claim that the adopted text which he must use makes it difficult for him to practice his belief, but the point is that it is his belief. No single dissenter has been found to the justifiability or feasibility of such a measure of professional expertness as will enable the teacher of mathematics to meet all five requirements. This means much for future improvement, for less than ten years ago these things were called "pedagogic vaporings."

II. *Social worth and psychological justifiableness are commonly accepted as criteria in both elementary and secondary mathematics. Utility and adaptability are the accepted tests of material to be selected.*

It has been but a few years since the high-school teacher, when reminded of the low value to modern society of some topic of advanced arithmetic, or of algebra or demonstrative geometry, arrayed himself at once in defence of the topic, and against the social criterion. The disciplinary virtues of the school subjects furnished him safe refuge from the impertinent criterion of social worth. Only a few years ago one significant teacher asserted that to study mathematics with concern for its utility greatly impaired the value of that study. Its worth to the student was in direct proportion to the degree of completeness with which he could divest himself of all notions of its usefulness. This attitude is today confined almost entirely to our colleges and universities. No important high-school teacher takes it. He reminds us at once that he and his significant colleagues have veered to the recognition of the following as a leading principle of curriculum-building for high-school mathematics:

Those truths, principles and methods should be included which yield the most effective control of the world of space and time and are at the same

time richest in their mathematical interrelationships. This dual type of material must be persistently attended to in teaching, though the outward aspect upon the world of reality should receive first and main emphasis in the high-school work.

This very rational, and comprehensively sane point of view seems to cover sufficiently the requirement of social worth, without sacrificing either scientific or psychologic soundness. May we not hope soon to see a high-school course of mathematical study really organized on this wholesome basis.

Psychological justifiability was hardly thought of as applicable to high-school teaching of mathematics a few years since. That the form and matter of mathematics should be materially and qualitatively modified from the type for adult presentation to adapt it to immaturity might be well enough for the elementary school. The high-school must take up the more virile task of mastering the logic of mathematical studies. This viewpoint has so much softened with recent years that one rarely hears nowadays of logic as an important aim of mathematics in the high-school. The good high-school teacher of today admits at once that the mathematical course of study which gives to the pupil at each maturity-level the particular type of knowledge and skill that are best suited to this level, with but minor regard to preparatory needs, is of supreme importance. This seems sufficiently to support my point that psychological justifiability has in recent years become an accepted fundamental principle of high-school teaching of mathematics.

III. *General acceptance of the doctrine of correlation in teaching early high-school courses, together with its natural corollary of general mathematics for these courses, is a third principle of recent acceptance by secondary mathematics teachers.*

A dozen years ago the doctrine of correlation was regarded as a fad by high-school teachers of mathematics. Those who may have been connected with the editing of high-school journals twelve or more years ago have not forgotten how it endangered the subscription list of journals for high-school teachers to champion the merits of correlation too earnestly. Teacher-training classes also at that time showed a decided

aversion to being treated too generously with this doctrine. One must search far and widely nowadays to find a high-school teacher who opposes correlation in teaching. Most of them are the warmest advocates of it, and stoutly profess to be making continual use of it. Mathematical times do change, and in the present instance the change has been for the better. An all-around correlation of all kindred high-school subjects, not merely with the mathematical branches among themselves, is indeed now quite the order of the day. In this regard we can exclaim with Galileo: "*E pur si muove.*"

Not all teachers have reached the point at which they are ready to accept general, or combined, mathematics as the necessary consequence of correlation, but most significant high-school professionals are ready to admit it as legitimate and under certain conditions, desirable. May we soon see the day in which high-school teachers of mathematics, generally, shall see that general mathematics for early high-shool courses is the only plan in full harmony with the spirit and educational psychology of our time. The extent of textual material and of the highly successful use of it in mathematical classes over the entire country have already taken from it the odium of mere local interest and significance which for some time it had. It will hardly be news to the up-to-date high-school teacher of mathematics that general mathematics is a most significant national movement. The success of this movement in high-school mathematics has been so marked that even many universities are employing combined mathematics in early courses. General mathematics is one of the leading recent high-school movements toward radical improvement.

IV. *Recognition of the urgent need for a general agreement as to what are the legitimate attainable purposes of secondary-school mathematics, to be used as a basis for a soundly reorganized modern course of study in both the junior and senior high schools constitutes our fourth pedagogical principle of recent acceptance.*

If this principle is a valid statement of a recently accepted view, it implies some rather significant things. In the first place, it implies the general recognition that thoroughgoing change is needed in the course of study in high-school mathe-

matics; that there is grave danger that the change may not be for the better; that it needs united counsel and support, and that now is the time to undertake reorganization. Perhaps some of these things arise from the fact that the plans already proposed, some of which are already embodied in texts, differ so widely in scope and nature as to make the average teacher wonder where the correct route may finally be found to lie. One can see in this statement also that it is desirable that the set purposes, whatever they be, shall be definitely attainable. Too often in the past the purposes set by the course of study or by the individual teacher, however worthy they may have seemed, have not been achieved by a sufficient number of pupils to demonstrate their legitimacy. If all these things are correct implications of a correctly stated principle, we may justly look for greater forward movement in high-school teaching of mathematics than the last fifty years have seen.

V. *Recognition of the fundamental and controlling place of the notion of functionality throughout the secondary mathematical curriculum may in the writer's opinion*, though he would be glad of a little higher certainty here, *be claimed as a recently accepted pedagogical principle by strong high-school teachers of mathematics.*

Recently it came to the knowledge of the writer that a leading high-school teacher whose success as a teacher has won the enviable distinction of functioning as an advisor to school officials of an important state, holds and voices the view that while collegiate people regard functionality as entitled to a dominant place in high-school mathematics, high-school teachers themselves see little or nothing in the idea. If this advisor is correct then the writer is incorrect in claiming this principle to have won recent acceptance from high-school teachers. The rather extensive poll the writer has made, as well as the written statements of high-school teachers in journals and elsewhere, lead him to believe that all high-school teachers who understand the principle, accept it. Since this could not by any stretch of imagination have been claimed half a dozen years ago, it is included here as entitled to the list of pedagogical principles recently accepted by high-school teachers of mathematics.

VI. *Ideal teaching of high-school mathematics consists in so implanting a definite quantum of mathematical knowledge and skill that the learner will, in the learning acts, develop right study habits and skill in applying knowledge.*

It is believed that the principle just stated lies in the background of the professional consciousness of all high-school teachers today, and it is certainly in the foreground of consciousness of many. A few years ago the problem of ideal teaching could not be stated so sharply as the work of the last few years have made it possible to do. Perhaps once it is stated in this simple form, it will be accepted by most high-school teachers after a little consideration. Whether generally accepted or not, however, it is of sufficient value as a definition and of sufficient acceptability to many significant teachers to merit its inclusion in the list of principles of recent acceptance.

VII. *Good high-school mathematics and good mathematical teaching are chiefly beneficial to the public-school pupil anywhere because, and in so far as, they beget and foster the habit of taking a rational attitude toward problematic situations, i.e., the habit of basing conclusions on the underlying facts.*

Every American voter needs this habit strengthened to the fullest measure of which he is capable. If the high-school teacher has not already accepted this as the leading purpose of American school mathematics to the ordinary citizen, its mere statement will be sufficient to gain his acceptance of it. It is the chief reason why we cannot afford to allow the mathematical element in education be reduced to the *minimum essentials.* We need the questions of the day settled not on the basis of its being accepted, or rejected, by one's favorite political leader or party, but solely on the basis of *the underlying facts.* No subject of the high-school curriculum is better adapted to this educational need than mathematics.

The writer must abstain from adding a few more vital principles of recent acceptance. It would seem that the more significant ones stated above will suffice for an affirmative answer to the question raised at the beginning of this paper.

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THE GEOMETRY OF THE JUNIOR HIGH SCHOOL.

The educational world is, like the industrial world, in a state of unrest. Many are seriously questioning the values of some subjects that have long been included in the curriculum and the educational literature of the day contains numerous suggestions for the relief of real and assumed educational ills. It is important that teachers and administrators should be as careful to discover wherein new theories are unsound as to discard the errors of the past. We must strive to keep our bearings in the midst of the confusion of the day.

Efficient teachers of mathematics will admit that the subject is not taught in an ideal manner. No subject in the entire curriculum is so organized or so taught. The mathematics of the school has not been sufficiently related to the mathematics of life outside the school. The roots of the mathematics taught in the school have not been sufficiently "imbedded in the soil of reality." It is not probable that a subject which the race has developed through the ages and which is intimately related to so many of the necessary activities of civilized man will be discontinued in the schools. The iconoclast who uses mathematics in the tabulations wherein he seeks to prove that mathematics is useless, cannot destroy the subject and he does not wish to improve it. Revisions, eliminations, and readjustments which are necessary will be made as the result of the judgments of those who are honestly seeking to improve the courses.

The rapid increase in the number of junior high schools is one of the most significant facts in recent educational progress. The history of education is replete with movements that have given promise of permanence but many of the reforms have proved to be ephemeral. The organization and development of the junior high school "seems sufficiently in accord with experience and with common sense to give some promise of permanence and hence to justify serious consideration."

A subject which is related so intimately to the needs and the interests of mankind as mathematics must be given a

prominent place in the curriculum of the junior high school. The subject may be so taught as to emphasize its application to the every day problems of life outside the school and, when so taught, it loses none of the charm which has attracted to it some of the best minds of the ages. The efficient teacher need not fear that mathematics will make a weaker appeal to boys and girls as a rsult of present adjustments, eliminations, and shifts of emphasis. The more firmly "the roots are imbedded in the soil of reality" the stronger and more attractive should be the appeal.

The course in mathematics in the junior high school should be a unit. It should be the best possible course for the pupil whether he does or does not continue through the senior high school. The course should prepare the pupil to meet his assured mathematical needs. The course should not be vocational in the narrow sense. It should awaken vocational interest but the junior high school period is not the time to develop skill in production. The course should develop the pupil in a broad way. It should be the basis of the mathematical training for his life career. It is not possible to anticipate all of the situations which will confront a pupil. The essential thing is, in the words of John Dewey, to give not the actual situations to be met later, "but the *power* to meet them." The mathematics of the junior high school should be so organized and taught that it will prepare the pupil to meet successfully the mathematical situations with which he is confronted while still a pupil in the school and with which he is mostly likely to be confronted outside the school. Moreover, the course in the junior high school should lead the pupil to a mastery of certain parts of the great subdivisions of mathematics. Bits of arithmetic, algebra, geometry, and trigonometry here and there are not satisfactory to either pupil or teacher. When the subject is presented in such a way the pupil is likely to leave the course with a confused knowledge of the whole and without that consciousness of mastery which is his right.

The first half of the seventh school year should be devoted to the study of arithmetic. This subject relates to the immediate mathematical interests of the pupil; it connects directly with the mathematics that has preceded, and it will enable the

pupil to maintain or increase the efficiency in computation which he has acquired. The work in arithmetic should be organized about certain large topics of practical value which challenge the interests and meet the needs of the pupils. Such topics as the following are admirably suited as cores about which the various phases of the work may be centered: The Arithmetic of the Home; of the Store; Industry, and the Bank.

The latter half of the seventh school year should be devoted to constructive and intuitional geometry. The subject is more concrete than algebra; it admits of more simple illustration; it relates directly to the arithmetic that has preceded; it challenges the interest of the pupil; and it may be made very practical. Some knowledge of geometric forms and mensuration is desirable in almost every walk of life.

The course in geometry suggested by outline in the following pages is now used in some of the best junior high schools of the country, and numerous teachers and administrators are witnesses to the fact that the course so organized may be taught in such a way that not only is the interest of the pupil challenged and held but the pupil is introduced to one of the great subdivisions of mathematics. He is enabled to understand and to appreciate mathematics as a tool, and to develop that feeling of power and of mastery which is his right and privilege. Such a course meets the dominant interests of the pupil and is adjusted to his development. It enables the pupil to develop power and the habit of interpreting more fully the quantitative relationships which exist.

Course in Geometry.

Many years ago when men began to study about forms they used to decorate their walls with pictures showing the shapes of objects. Later they drew plans of homes and temples and drew pictures of animals and human beings. As land became more valuable they became interested in measuring fields and building materials, and it became necessary for them to consider position and to locate places on the earth's surface. From early times men have been interested in ideas of form, size, and position. The three questions which we frequently ask about an object form the bases of constructive and intui-

tional geometry. These three questions are: What is its shape? How large is it? Where is it?

The pupil is already more or less familiar with many of the common geometric figures such as the square, triangle, circle, arc, and cube. He should learn early in the course the various kinds of angles, triangles, quadrilaterals, and other common polygons and he should be taught to observe these forms in the school room, on the school-grounds, and elsewhere in his daily activities. He should be taught to use simple drawing instruments such as the compasses, the ruler, the protractor, and the right triangle in the construction of the various kinds of triangles and polygons and in the making of various geometric patterns and designs. He should be led to discover by experiment and observation the truth of some of the most important propositions to be proved later in demonstrative geometry. He should be taught to think clearly about form, size, and position, and to *see* mathematics in nature and in objects of human construction. The following outline will be suggestive:

Outline of Constructive and Intuitional Geometry for the Last Half of the Seventh School Year.

The Geometry of Form. What Shape Is It?

Angles and triangles; quadrilaterals; common polygons.

Use common drawing instruments, such as the compasses, the ruler, the protractor, and the right triangle.

Construct various kinds of triangles and discover the angle sum of any triangle.

Construct perpendiculars and bisectors.

Construct angle equal to a given angle and triangles with various parts given.

Construct parallel lines and develop the principal theorems related to angles formed by a transversal which cuts parallels. Use the protractor and paper cutting.

Make various geometric designs by means of simple constructions.

Draw to scale. Apply to the problems of the builder, the farmer, the engineer, the designer, and the geographer.

Similarity of shape. Similarity in photographs.

The pantograph. Symmetry.

Provide numerous practical problems including much outdoor work involving the applications of the preceding. Introduce each topic in such a way as to challenge the interest of the pupil and to appeal to him as worth while.

The Geometry of Size. How Big Is It?

Introduce numerous practical problems involving estimates and measurements of heights, distances, and areas. Estimate dimensions of school room, school grounds, etc., and check estimates by measurement.

By use of congruent triangles, measure the heights of such objects as trees, buildings, and flagpoles, and the width of rivers; and check results. Encourage pupils to suggest problems which appeal to them as practical. The pupil is delighted to find that he has sufficient mathematical knowledge to determine inaccessible heights and distances. He has the knowledge and he should feel the power.

Introduce the study of areas by the use of squared paper.

Develop the formulas for the areas of the common plane figures and apply to numerous practical situations such as those which most commonly confront the pupil, the farmer, the housewife, the engineer, the business man, and the artisan. Continue the emphasis on outdoor work.

Introduce ratio and proportion and use proportion in the determination of heights and distances involving similar figures.

The Geometry of Position. Where Is It?

Introduce this subject by problems, such as locating the proper position for second base on a baseball diamond after the other bases have been located.

Challenge the interest of the pupil by introducing problems dealing with attempts to locate buried treasures.

(*a*) Develop and apply practically the method of determining points equidistant from two given points; (*b*) distance of a point from a line; (*c*) points equidistant from two lines; (*d*) points at a given distance from a given point.

The applications of the geometry of position are numerous and interesting. The pupil may be led to appreciate that he is acquiring knowledge that is necessary to the solution of many problems of a most practical and interesting type.

Experience seems to indicate that the arithmetic and the constructive and intuitional geometry of the seventh school year should be followed during the first half of the eighth school year by algebra. The introductory course in algebra should no more be formal than the introductory course in arithmetic should be a formal course in computation. The algebra should be such as every boy and girl should become familiar with at this time. It should not be burdened with technical phraseology but it should be utilitarian in the largest sense. Some knowledge of the formula is needed in reading many books and articles, the graph is used in many lines of business, the equation is helpful in manipulating formulas, and negative numbers are so commonly used as to be a necessary part of the equipment of every reader of current literature as well as of scientific books.

During the last half of the eighth school year those practical topics of arithmetic for the study of which the pupil's maturity has prepared him should be studied. Among these topics the following should be included: The Arithmetic of Trade, Transportation, Industry, Building, Banking, Home Life, Farming, Civic Life, and Investments.

It is not improbable that the ninth school year will eventually be the last year of *required* mathematics. It is very desirable that the pupil have some knowledge of formal mathematics.

During the ninth school year most of the time should be devoted to the study of elementary algebra and geometry. Algebra should be placed first as the student is familiar with it and needs to use it in his other mathematical work. By omitting the non-essentials, the pupil can complete algebra through quadratics during the first half of the year. The trigonometry of similar triangles may be introduced.

There should be a gradual introduction to demonstrative geometry. Independent deductions should precede formal proofs and a large number of practical exercises should follow

each proposition that is studied. Extreme formality of treatment should be avoided and originality, clearness, and conciseness should be emphasized. The formality of some introductory texts in demonstrative geometry tends to discourage many pupils. The pupil should recognize the necessity for a proof and he should understand the nature of a geometric proof. He should be taught to prove a number of the most important theorems such as the theorem of the angle sum and the square on the hypotenuse. The exercises should be simple in nature but they should encourage that independence of mind which is more valuable than a knowledge of a conventional number of formal propositions.

Experience has shown that pupils of the ninth school year enjoy demonstrative geometry when it is properly motivated and presented. Pupils do not find the subject difficult. Many of the important theorems of the first two books can be proved and applied by the pupils. Practical applications which are within the understanding and the experience of the pupils are numerous. The initiative of the pupils is encouraged.

The student who has followed such a course as is here outlined has studied the elements of mathematics and is in a position to know whether he cares to pursue the subject further. The door of the science has been opened to him, and he has entered. He is in a position to proceed further, if he cares to do so. The course is arranged in psychological sequence and it is utilitarian in the best sense. The course is so organized that it gives the pupil that feeling of accomplishment and mastery which is essential. It articulates the various subjects and forms a natural transition between the mathematics of the lower grades and that of the senior high school.

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ALGEBRAIC MAGIC SQUARES.

There comes a time in the school year, usually during the spring term, when the mathematics teacher becomes convinced that as far as algebra is concerned, he might just as well be teaching so many "wooden Indians." Those pupils, who are not wholly in a trance, are surreptitiously fondling a baseball glove, while x's and y's pass by unheeded. The teacher's first impulse is to give every one a good shaking in a frantic attempt to close the ever-widening gap between the intellectual capacity of his pupils and the intelligibility of his subject. He realizes something must be done at once, if his class is to learn any more algebra that year.

The introduction of graph work into the first year of the mathematics course has done much to solve this problem. A baseball graph in red and yellow colors showing the standing of the local team and the leader in its league, helps to convince the boys that there is something human about their algebra teacher, while a discussion as to whether a regulation baseball diamond is a perfect square, to which every boy enthusiastically brings the latest baseball guide, and the teacher the Pythagorean theorem with its horrible name artfully concealed, really persuades the boys that mathematics after all may have a vital bearing on the big interests of life when no amount of engineering problems could possibly do so.

For the writer, the particular bugbear of the school year is the unhappy meeting of his large classes of boys, and the two weeks just before the spring vacation. That the boys have "gone stale" is evidenced by their apathy, and more alarmingly by their restlessness. The graph work may be used to advantage to tide over this period, but to the pupil returning from his vacation, it has become an old story. Much better results have been secured by postponing this work until the boys have had a chance to play a little baseball, and by introducing magic squares into this pre-vacation period.

As their name implies, these squares were believed to have magic properties in the early days of history by various peoples

who interested themselves in numerical studies. In fact a magic square on a small metal plate hung about the neck was considered a powerful means of warding off sickness. Fig. 1 is an example of the simplest kind of magic square.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Fig. 1.

The middle cell of the top line is occupied by the first number, and the successive numbers are placed in their natural order as follows: when a number is placed in the top line, the next number is written in the bottom line in the nearest column at the right. Whenever it is possible, the numbers are sloped upward to the right. When a number is placed in the column at the extreme right, the next number is placed in the column at the extreme left in the line above. When none of the preceding rules can be followed, place the next number in the cell immediately below, and go on as before. When the square is completed, the same sum will be obtained by adding the numbers in the columns, lines and even the diagonals. In the example given above the sum is 65.

Why such a square should excite more than passing interest in first-year pupils, the writer will not attempt to say, but the fact is that the sight of the teacher sprinkling a few numbers here and a few numbers there in this gridiron arrangement, and then getting the same sum from the vertical, horizontal, and diagonal addition, is electrical in its effects on all the pupils and particularly on those whose mental numbness is strongly pronounced. Once their interest is caught, the pupils are encouraged to find all they can about magic squares in

encyclopedias and the books on mathematical recreations in the public libraries. As the above rules are for magic squares of an odd number of lines and columns, they are not satisfied until they have found a set of rules for the more difficult even number of cells. The work is as perennially interesting for the teacher as for the pupils, as each year some

238	143	234	153	228	241	147	231	154	146
148	214	161	221	171	166	211	173	215	235
224	165	204	181	209	177	175	203	218	159
144	220	183	191	186	197	192	200	163	239
158	219	205	193	196	187	190	178	164	225
232	160	176	184	189	194	199	207	223	151
227	170	210	198	195	188	185	182	213	156
157	216	180	202	174	206	208	179	167	226
150	168	222	162	212	217	172	210	169	233
237	240	149	230	155	142	236	152	229	145

FIG. 2.

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unusually ingenious form of magic square, like Fig. 2, is unearthed by some one in the class from juvenile magazines at home. This example illustrates what is known as a bordered magic square, as the borders may be successively removed without destroying its "magic" properties until it is reduced to four columns and four lines. The numbers in each of the inscribed crosses have the same total as a line or column of the complete square.

Only one period of class time is necessary to give this work a good start, and for the rest of the time it may be carried on along with the regular course. The question naturally arises as to whether the fascination of this work does not cause the pupils to neglect their algebra. Such is not the case. The interest aroused by the magic squares secures a mental alertness which transfers to the other work. Furthermore, it is an interesting and very significant fact that most of the pupils are sure to become restless and even a little worried if the regular algebra work is wholly put aside for any length of time.

A happy combination is secured by assigning a magic square in addition to the usual home-lesson; first a square of 25 cells beginning with the number 1, then a larger number of cells and larger initial numbers. The addition of every line, column and diagonal must be insisted on as the only true test of a genuine magic square. Quick methods of addition can be very profitably introduced here, and even a passing mention of lightning calculators and their feats will do much to heighten the growing interest.

Soon the boys are engaged in a contest to see who can make the largest magic square. As a rule they try to secure extra credit for this work which may very properly be allowed. At times the writer has had misgivings as to whether magic squares with a hundred cells on a side and drawn on a piece of wrapping paper two or three feet square did not signify an interest which had been carried to an unreasonable extreme. However, as there are no injurious reactions, and as the boys seem to get so much fun out of it, a sympathetic attitude may well be shown by the teacher so long as the pupils are willing to do the necessary addition.

When they seem reluctant to do this, the time has come to turn to algebraic squares. Indeed, sooner or later, some one is sure to ask if there is such a thing as an algebraic square. A search in the library fails to reveal any description of one, so the pupils are invited to make them up themselves. The first attempt is fairly sure to be the usual magic square with some one letter placed after each number. Then they are asked to make a square in which both plus and minus signs are used. There is no rule given them, but they must make it as

best they can by guessing and trying. Squares whose lines and columns have the same sum are accepted for credit, even if the diagonals do not. This rule places the task within the

$8x + 5$	$x - 9$	$6x + 1$
$3x - 5$	$5x - 1$	$7x + 3$
$4x - 3$	$9x + 7$	$2x - 7$

FIG. 3.

mental reach of every member of the class, and it is really touching to see the time and energy that the lowest tenth of the class will put in on this work. It is hardly necessary to say that the formation of a square of even nine cells of this

$4x$	-3	x^2	-1	$2x$
-1	$2x$	$4x$	-3	x^2
-3	x^2	-1	$2x$	$4x$
$2x$	$4x$	-3	x^2	-1
x^2	-1	$2x$	$4x$	-3

FIG. 4.

kind gives considerable practice in algebraic addition. From monomial cells the next step is the binomial cells which are not much more difficult. Fig. 3 is an example of this kind of square. Polynomials may also be attempted, but are rather unwieldy.

Multiplication magic squares are next in order; that is, squares whose lines, columns and diagonals have the same products, as in Figure 4.

More complicated magic squares like Figs. 5 and 6 are encouraged and accepted for extra credit, but are not required of the whole class. All of these examples in algebraic magic

$18x^2 - 20x \div 2x$	$16x^2 - 24x \div 8x$	$21x^2 - 24x \div 3x$
$16x^2 - 20x \div 4x$	$12x^2 - 14x \div 2x$	$40x^2 - 45x \div 5x$
$15x^2 - 18x \div 3x$	$20x^2 - 22x \div 2x$	$18x^2 - 24x \div 6x$

FIG. 5.

squares are the work of Boston English High School boys. The sums of the products or quotients in each of these figures are the same.

$2(x-4)$	$3(2x+3)$	$5(4x-1)$	$7(8x-9)$	$8(6x+2)$
$7(8x-9)$	$8(6x+2)$	$2(x-4)$	$3(2x+3)$	$5(4x-1)$
$3(2x+3)$	$5(4x-1)$	$7(8x-9)$	$8(6x+2)$	$2(x-4)$
$8(6x+2)$	$2(x-4)$	$3(2x+3)$	$5(4x-1)$	$7(8x-9)$
$5(4x+1)$	$7(8x-9)$	$8(6x+2)$	$2(x-4)$	$3(2x+3)$

FIG. 6.

The interest of those pupils who are unable to make the more difficult squares can be kept up by requesting them to check the accuracy of the magic squares when they are put on

the board. This can be done quickly by requiring the first row of the class to find the sum of the top line of products or quotients, the second row the second line, etc. This work is eagerly pursued by most of the class, as most of them cannot be convinced that a task, in which they themselves have failed, can possibly be performed until every line, column and diagonal has been checked up.

The following simple rule for a square of five lines and columns is finally discovered by the pupils. The top line is made up of cells containing indicated multiplications or divisions. This line of cells is then, repeated in the next four lines in the order shown in Fig. 7. The fourth cell in the first line becomes the first in the second line and so on.

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

FIG. 7.

Finally when the last algebra period on the last day of school before the spring vacation arrives, the teacher rubs his eyes to find even some of his mathematical "black sheep" still lingering to ask questions about magic squares, and he becomes convinced that while some may be taught by a direct attack, others must be taught by stealth.

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THE OUTLOOK WITH REGARD TO SCHOOL MATHEMATICS.[1]

Few teachers, probably, realize that they are living in a period of great mathematical development. New phases of old problems, new problems and new methods are continually demanding some new modification in mathematics or else out and out new methods in mathematical procedure.

Critical studies in physics and astronomy have in recent years called forth a new theory of relativity which is essentially mathematical. No one but the mathematician could compass this problem. The result is a serious revision of long standing theories.

Much attention has been given lately to the study and discovery of empirical formulas for the concise statement of physical, economic or social phenomena. Recently several treatises have been published in this field. Only a few weeks ago the writer was called upon to assist in the determination of a formula for simplifying the cost estimates on contracts for iron castings. One of the prominent engineers of the city was working out the problem.

Just the other day a professional educator came for advice in regard to a formula which he had arrived at empirically. He wished to eliminate the element of probability from the scores of pupils in a certain test. And so it goes on. It may be of interest to add that the mathematical side of both problems involved the use of certain parts of the traditional courses in algebra.

The recent income tax legislation has called forth new interest in statistical methods in several great industries to such an extent that private schools of statistics designed especially for the purpose have sprung up. The writer had a conversation with the principal of one of these schools a few days ago. Here again the special course is built on a modicum of standard mathematics and general statistical method. Much use is made

[1] This paper is part of an address given before the Mathematics Section of The Education Association of Western Pennsylvania, February, 1920.

of graphic representation and the use of calculating tables, both of which are begun in any well-designed course in algebra.

These examples are significant. They point not to the death of mathematics but to a changed emphasis and a modification in the content of school courses. Some topics must be further pared down to make room for the new problems and their attendant theory. Nothing vital and universal will be nor can safely be given up. Typical mathematical procedure will still be essential. Instead of an even emphasis on all the pages of an antiquated text, as the writer has observed in some instances, there must be a more critical selection of the universal fundamentals for greater emphasis and a new set of applied problems suited to present progress. This does not mean the elimination of all old problems but a closer selection. The time spent in first-year algebra factoring such expressions as $x^{16} - y^{16}$ or $a^{21} - b^{21}$ can be more profitably spent in further drill on simpler cases and in applications. It means more attention to certain graphical methods, especially to the interpretation of graphs in connection with statistical data and simple forms of functions. Some study of averages, such as arithmetical mean and weighted mean are easily within the grasp of high-school pupils. In the more advanced classes the idea of chance and probability can be given with advantage. Probability enters a large number of scientific and social studies. As an illustration, consider a test questionnaire of twenty-five questions, each with six suggested answers, only one of which is correct. Ask the candidate to mark the correct answers. By the law of chance he will get one sixth correct if he knows absolutely nothing. One might stand off and shoot at the questionnaire with a shotgun and be sure that one sixth the answers hit by shot will be correct. It is sometimes desirable to eliminate this chance element from the test. This is a mathematical problem.

Some General Observations.

1. The tendency of all organized society is toward more scientific study of relationship in economic, social and natural phenomena.

2. Scientific study invariably tends to measurement and functionality between elements or parts of observed phenomena.

3. Measurement and function study are preeminently the field of mathematical study.

4. Some knowledge of mathematics is desirable and necessary to the equipment of any student of natural and social phenomena.

5. The mathematical knowledge necessary for the above purposes must include some scientific training in typical mathematical methods and some skill in mathematical technique.

6. There must be a certain ability to apply the mathematical methods most adapted to any given particular situation.

7. The rapid changes in social and economic affairs in the last few years have opened fields of study to such a degree as not found in any similar period in history wherein mathematical training is needed. This means a most unusual opportunity for teachers and students of mathematics.

8. It probably is not mathematics that has been criticized but rather the too conservative disciples of the science in their administration of the content and methods of elementary courses. The process of adaptation has been too slow.

9. The mere lesson hearer and the text book slave must be eliminated or rejuvenated. Teachers of mathematics must have a contact with affairs, not shop problems alone, if our subject is to perform its natural and possible service to civilization.

We are in the throes of a spasm of educational measurements. We are attempting to measure almost every thing in school. This is only a chapter in the mathematization of all science. Often we have a desire to measure and do not know what or how to measure. This desire may be a sign of good intentions and scientific spirit. In attempting to measure we may learn what to measure and how to measure it. We are indeed making some progress in measuring the more obvious attainments in mathematics, viz., formal technique. The tests in algebra measure with some certainty the pupil's attainment in formal manipulations. Technique is essential to any use of mathematical applications in science. But technique alone will generally not be sufficient to ensure the use of mathematical methods in practical situations. We are therefore far from measuring the whole of the results of good teaching or

the ultimate values of mathematics as an educational instrument in a complicated and scientific civilization. We have not gone far enough to know definitely just what to teach and why and how. We should not however be discouraged for the problem is a large one and cannot be solved in a day or in a year. It is a problem for a generation. We can and must contribute our part toward the solution and our children will see it finished.

An able psychologist and mathematician has laid down some definite general aims in teaching mathematics:

1. Elementary manipulations.
2. Analysis of a problem or situation.
3. Symbolization of data and relations.
4. Manipulations necessary to a solution.
5. Interpretation of results.
6. Power to extend the methods to new fields.

We should as soon as possible take steps to measure accomplishment under these heads. It is conceivable that two or more of these may be tested by the same questionnaire. It may be necessary to further subdivide the heads to make satisfactory measurements. We must first determine what distinct general steps are involved in the solution of a problem. It is probable these tests will be more difficult to formulate than the technique tests.

Tests made during the progress of a course are no doubt valuable, but there should be general and broad tests at longer intervals covering the entire work of two or more years. Some of the richest generalizations and transfer values do not come until the end of an extended term of study or even after the subject has been laid aside.

The psychology of specific attainment has held the stage for some time. During this time constructive scientific psychology has been "sawing wood." There has come a doctrine of generalized habits. The attacks of the destructive psychologists have been halted and we are entering an era of better considered principles which promise much for the theory of education. The writer has stated in another paper: "Psychology cannot teach us how to teach but can and must furnish the basis on which we are to build the theory and practice

of teaching." The doctrines of generalization and of general habits seem destined to play an important rôle in educational theory during years to follow. Our tests and measurements from now on must recognize these results of psychological research. Those who have "buried formal discipline" will not necessarily be qualified to lead us in the next steps.

Mathematics will not be entirely nor even almost eliminated from the curriculum. But a new type of mathematics which is better adapted ultimately to our recent developments will take the place of the traditional courses in the secondary schools. Instead of the mature and finished treatises of Euclid in geometry and a similarly complete algebra we shall select more carefully the essentials of theory and technique and turn to newer applications and to different estimates of value. The geometry of Euclid is now only a small part of the available material in geometry for school purposes. The traditional algebra is only a part of the available analytic material. We shall continue the pruning process on one hand while on the other we shall graft new scions into the old trunk. The species of the fruit may not be different but the variety and the flavor will be different. Less elaborate formal exercises and more graphic representation, some treatment of averages, theory of investment, etc., will be included.

In geometry many complicated and little used theorems may well be omitted to make room for drawing to scale, new applied problems and some modern developments in geometry.

The new course in mathematics will be more of a composite, broader and cast in closer touch with the progress of civilization. This modification of course refers to the school courses in mathematics. The colleges and universities can never dispense with abstract mathematics. For here are formed the foundations of scientific advancement and research. Certain groups of students may with advantage be given more concrete courses.

All these changes must be made while still retaining the typical fundamental truths and methods of mathematics. There must remain enough of theory, rationalized and logically presented, in order to secure the power which must come and can come only from mathematical study. The course must retain such distinctive characteristics as to make the subject

self-perpetuating and universal in human thought. To reduce mathematical instruction to the incidental use of formulas in engineering, chemistry and accounting would be a serious error in educational policy.

One writer has recently stated that "Before 1700 there were six important scientific discoveries or inventions. Since 1800 there have been fourteen: The first six are writing, numeral system, compass, printing, telescope and barometer. The orthers are steam engine, railways, steam navigation, telegraph, telephone, friction matches, gas lighting, anæsthetics, antiseptics, electric lighting, phonograph. In current times we may add the cinematograph, wireless telegraphy and telephony, electric navigation, internal combustion engines, aeronautics, steel production, submarine navigation.

It is not too much to say that without mathematically formulated sciences our civilization would never have been possible. If the study of mathematics both high and low were to be stopped now, a relatively short period would see the whole civilized social and economic structure collapse.

What is practical mathematics? Who shall say? The highest theoretical mathematics of fifty years ago is now applied to science in such a way as to affect our industries. Who shall be responsible for asserting that the highly abstract branches of mathematics of today shall never find application in civilized society? Mathematics is one of the long range eyes of science and industry. Mathematical theories of physical phenomena have pointed the way to great inventions long before the so-called practical man realized their possibility. Mathematics has truly a prophetic eye. While there is not time for more than general statements here, there is not lacking all desirable historical evidence and specific cases to support the claims made. The instances are not exceptional but the general rule in scientific and industrial development.

We hear much about the "practical" in elementary and secondary mathematics. Even the college teacher is beset with the query "what is this or that piece of mathematical work going to net me in my profession?" You will possibly be surprised that students of engineering are the most frequent in such inquiries. Strange it is, too, when without mathematics the profession of engineering could not exist. The

advocates of practical mathematics range all the way from the pure empiricist, who cares only for a few formulæ used in the routine of his little world of business or profession, to the "projectionist" who must have a "project," real or imaginary, for every new mathematical step in the whole course. The first are evidently too narrow to perpetuate even their own interests and could not reproduce their formulæ if lost nor can they intelligently extend their mathematics beyond their present attainments. They cry against what has made them possible and what will keep them going. The last are so absorbed in their projects that there is danger that they may overlook the true spirit of mathematics and submerge it in a mist of material interests. There must be a reasonable mixture of the theory and its material connections. But to reduce the theory to a mere incident in the development of a material interest will defeat the purpose of the materialist himself.

Our educational system must always furnish a surplus of possible investigators in pure mathematics by giving a considerable number of pupils an opportunity to try out the field of mathematics. What is said of mathematics may be said of other instrumental branches. Whenever the world catches up, so to speak, with mathematical research, by having made use, in science or industry, of all available mathematical knowledge (and this would not be impossible if some educators had their way), civilization must come to a halt until further researches are made. Any nation that permits its educational system to omit or neglect a subject so fundamental and universal will suffer in the next generation. Modifications and adaptations to meet new situations we must admit and encourage; elimination, never. We as a professional group will be heard in proportion to our capacity and readiness to meet the actual problems of civilization successfully.

In order to facilitate the constantly present need of adjustments in our subject, it is highly desirable that teachers of mathematics have some side line whereby they may keep in contact with affairs. This will be a source of support and encouragement and a reservoir from which to draw live problems to illustrate and enforce the teaching.

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MATHEMATICS IN STUYVESANT HIGH SCHOOL.

For the past three years Stuyvesant High School has been using for the first four terms a course radically different from the old established arrangement of a year of algebra followed by a year of geometry.

The controling aim of the new course is: To give the pupil the mathematics that is best for him whatever the length of his school course. This means making provision for the boy who can remain in school only ten weeks, giving him what he needs most, and at the same time not neglecting the proper training of the boy who will go to college. Moreover, it is desirable that these two boys be taught in a single course rather than in separate courses, since the former boy may be persuaded to continue in school and go to college.

Suppose a pupil can remain in school only ten weeks. What mathematics can we give him that will be of most value to him? If he leaves school and goes to work, he may desire to go to an evening school and study mechanics or physics. He may desire to read a mechanic's handbook and know how to use the formulas in his daily work. Whatever he may do for living, he will find it very useful to know how to use the practical parts of elementary mathematics. This includes:

1. The ability to make a formula, use it, solve it for any letter, and interpret the result;

2. The knowledge of how to measure ordinary plane and solid geometrical figures;

3. Practice in computation, including the checking of all operations, until reasonable accuracy is secured;

4. A natural introduction to algebraic work through the motivation of the real problem;

5. Preparation for demonstrative geometry through familiarity with the material of the subject found in the mensuration of practical problems.

A graded series of real problems captures the interest of the pupils at the start and holds it throughout the course. From the first lesson in which the boy measures the height of the

room in which he is sitting, by the use of a simple isosceles right triangle cut out of pasteboard and from this makes his own formula, to the more difficult problem of laying out a baseball diamond, involving a quadratic equation, the growth in correct mathematical habits is rapid. The student's interest in doing things at the same time that he is learning to use letters as numbers is much greater than under the old method.

The problems used are of such a general nature that they can easily be taught in any type of high school or in the last year of the junior high school. At the end of ten weeks the student has completed all of the mathematics that can be justified on the ground of possible utility. The next best thing has seemed to be an extended ability to use the equation in concrete problems. Hence the second ten weeks is devoted to the equations of elementary algebra with applications to concrete problems through quadratics, omitting most of factoring and fractions, all of exponents, and all of radicals.

For the second term, plane geometry with its training in forms of reasoning seemed more useful than the abstract part of the algebra. Hence this term is devoted to demonstrative geometry. In term three, geometry is finished and the State Regents' examination passed. Term four is used in completing elementary algebra including the Regents' examination. Beyond term four mathematics is elective and includes the usual subjects required for colleges together with a course in surveying and the use of the slide rule.

The results of this course may be summarized as follows:

1. There is much more mathematical insight and joy in the work than by the old method.

2. From a series of real problems properly organized, better training can be secured than from the ordinary abstract algebra.

3. The Stuyvesant Plan is an easy and natural method of introducing the pupil to the use of letters as numbers in algebra.

4. It is safe as far as Regents' examinations are concerned, as our records show.

5. Stuyvesant has tried to make the teaching of mathematics democratic,—that is, of the most use to the most people.

Course in Surveying and the Slide Rule.

This course is given in the fourth year, five periods a week for twenty weeks. Every student electing the course is obliged to take three hours per week of field work in the afternoon. This subject may be elected by boys who are taking trigonometry or have completed that subject.

The content of the course is equivalent to the first course in surveying in Columbia, and is so credited by that university. The work in the slide rule occupies about two weeks of time and includes a thorough drill in the operations performed on that instrument.

The text-book in surveying is Tracey's "Plane Surveying." For the slide rule, the manual published by Keuffel & Esser, New York City, furnishes sufficient material for the work of the course.

At the conclusion of this course a student is fitted for the usual field work, mapping, or computation of a surveyor's office. From this course entrance is easy into the City, State and Federal Civil Service in the capacity of chainman, rodman, or computer. Many of our graduates have found that his course has fitted them for good positions in which they earned a reasonable salary at the start and which served as stepping stones to very desirable engineering positions.

During the war, from this course, Stuyvesant High School furnished expert computers for service at the Sandy Hook Proving Grounds, where range tables for the big guns were constructed. In this work the slide rule was especially valuable.

This course in the use of the transit and the slide rule is eagerly elected by every student who can get it into his program. It is of immediate financial value to the boy who is not going to college, and it is credited as advanced standing to the student who goes to college.

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ARTICULATION OF JUNIOR AND SENIOR HIGH-SCHOOL MATHEMATICS.

Articulation while only a borrowed and a figurative word still implies the joining of things more or less distinct though as closely "articulated" as the arm and the body in human anatomy.

A better picture of the ideal relation of the junior and senior secondary schools, if taken from anatomy, is the way our muscles and tendons unite. Innumerable microscopic strands of connective tissue from innumerable microscopic muscle fibers are extended to a point beyond which there is no longer muscle tissue but only tendon—yet the tendon reaches and is attached to every part of the muscle.

So we may picture the ideal relationship of the senior and junior schools—the aims, purposes and courses of study so closely bound that even specialists cannot tell where one leaves off and the other begins. The separation in years and in buildings we should aim to make of no more actual significance than is the length of the sleeve to the arm muscles it covers.

The junior high school is primarily a finding and a sorting school—here the tastes, aptitudes and capacities of pupils are to have an intellectual try-out, based upon real *first hand experience* with some of the school work that lies just ahead.

No longer must children make their selection of a high school or of a high-school course a matter of chance, of faith or of a blind obedience. No longer must children *enter* a high school *first* and find out what is *taught there, afterwards.*

There should be in the junior high school—which is a "finding and sorting" school—courses of study that are finding and sorting courses.

The lines of work that lie just ahead are not merely studied *on the map* as formerly, but the pupil actually travels *in person* along each of the main lines of advanced study—if but for a very short distance—yet far enough in most cases to show the pupil, his instructors and his parents, where that pupil's talents and aptitudes lie.

I.

First among the essentials for the perfect and harmonious cooperation of the two schools—or the two phases of one school as they really are—is the planning of courses of study (or if we modernize our terminology, "curricula") that enable the pupil to make his successive steps of progressive differentiation and specialization in his work come as the result of actual first hand experience in his class room.

In the field of mathematics you ladies and gentlemen have shown the way, to the teachers and supervisors of the other major divisions.

Is it unreasonable to expect that within a very few years we shall find in the junior schools a plan consisting of one, two or three years of work along these principal lines:

1. General Introductory Mathematics.
2. General Introductory Natural Science.
3. General Introductory Social Science.
4. General Introductory English.
    a. Magazines. *b.* Newspapers. *c.* Classics.
5. General Introductory Foreign Language.
6. General Introductory Art.
    a. Drawing—Manual Training.
    b. Music—Vocal.
7. General Introductory Body-Training.

It is unnecessary to call attention here to the absolute necessity of having these courses or curricula that are anywhere to be locally administered—planned either by one mind, or by a group of minds in conference, to the end, that

First, each course shall *first* of all embody the principles of *unity in purpose*, and *grading in difficulty*, and that

Second, each shall be so far as is humanly possible made up of selected bits of *reasonable adolescent experience* rather than of selected excerpts from secondary textbooks.

Again you have blazed the trail, mathematics.

II.

A second essential to a perfect and harmonious union of the two phases of secondary school work is *unity of supervision.*

It has been repeatedly urged in reports and surveys that

junior high-school administrators should be experienced as actual teachers in both high and elementary school work. Where is is possible to secure supervisors of this type, no better guarantee of unity in supervision is necessary.

Our difficulties here (and they have been and still are most discouraging) have arisen from a conscious or unconscious *partisanship* of the supervisor, based upon his previous experience as a teacher. There is no question but that such a partisanship works injury to the junior pupils—no matter on which side the supervisor's preferences lie.

The suggested appointment of subject-supervisors, in the major lines of work—mathematics, English, social science, natural science, etc., is open to the same objection. Where are these supervisors to be found who combine an appreciation of their *subject* with an appreciation of an elementary school child's *mind?* The supervisor may know biology but does he know boys?

Time however will cure and is curing this defect, as the places at the top become filled with those teachers who are finding their way to promotion through the junior schools. We are discovering that the teacher that is able to teach successfully in a junior school, is equally able to teach successfully in a senior school.

As a second factor in supervision—the uniform compulsory ninth year examination has been proposed.

Such a proposal, while it may seem harsh, still if modified by mutual agreement to cover a series of examinations drawn by both junior and senior high-school teachers in conference, has much to merit consideration. This certainly would be one way of forcing continuity of instruction—and might if not abused, lead to a better articulation of work, but on the other hand it might as easily lead to all the well-known abuses—cramming for the examinations—teaching *for* the *subject only*, and *not* for the *pupil*—frightening away the less persistent and often killing off the more able along with the less fit.

Promotion by subject from the one school to the other is of course highly desirable when possible—but frequently this is not possible because ninth year subjects are not repeated in many senior high schools.

Unity in plan and unity in supervision are after all but means to an end. Even with perfect unity so far secured we have not yet reached the pupil—nor do we reach him until we enter the class room in the person of the class-room teacher.

We must admit that *no plan* and *no supervision* can do much more than to make this desirable unity or continuity of instruction *possible* and *attractive*.

To make the *possible* become the *actual*, the cooperative efforts of the teachers in both schools is absolutely required.

The one greatest enemy of the perfect union of the junior and senior high schools is a lack of acquaintance of the class-room teachers in the one with those of the other. From this ignorance spring distrust, and recriminations that lead us only into greater estrangement.

Whether or not I have the honor to be the first to discover one means of curing this ignorance and distrust in class-room instruction, I have at least found a remedy as simple and easy of application as it is efficacious. The actual, living unity and continuity in and between senior and junior schools can be secured neither by printed plan nor by careful supervision as successfully as by *compelling* the teachers of the two schools to become acquainted with each other's work. In a word this remedy is to make compulsory and without the possibility of escape, a personal, *first-hand* acquaintance of the *work*, the *aims*, the *methods*—of the junior and senior high-school *class-room teachers*, the one, with the other.

No other thing, can replace this mutual knowledge by actual exchange of visits—no lectures, addresses, articles or conferences can be substituted for the visit *in person*. No other thing can claim to approach in importance this mutual observation of work, and mutual study of purposes.

Unfortunately in many communities this exchange of visits will never be accomplished unless it is laid down as an unavoidable duty. Hence, my insistence on the compulsory and serious nature of this exchange.

From this exchange of visits comes first a better appreciation by the teachers of both schools of the subject matter to be taught in the other school. This of necessity will lead at first to a greater conformity of the junior school, to the senior

requirements. The first and foremost thing that the junior high-school boy or girl must do is to *survive* in the senior school. Unless there is "survival" there can be no continuity in fact.

However well taught and well trained in other lines a junior high-school pupil may be, unless that pupil is able to sustain himself in the entering term of the senior high school *all is lost.*

The junior high-school teacher who is preparing pupils for the tenth school year must be compelled—not merely invited—compelled to observe high-school work *in that year*, must be *compelled* to study the situations her pupils will be ultimately forced to face. These visits must not be optional, perfunctory, casual but *required* as of as great (or greater) importance than any work she may undertake in her own classroom in her own junior school.

During the initial years of any junior high school's existence, and thereafter until a high degree of continuity is secured, not less than one day each month—and preferably more at the beginning—should, by official direction, be *required* of each junior teacher for personal observation and study in the senior school.

But though the burden lies chiefly upon the shoulders of the junior high-school teacher there is still some obligation on the other side. If by visits and personal observations the senior high-school teacher becomes convinced that the junior pupils are really being well taught (though still in some respects not as he himself would teach them), there will come conviction that if these pupils do not at first make a complete adjustment, possibly the fault may not be *wholly* that of the junior school. The senior teacher as a result of his visits will be led to see that possession of a college degree after the completion (many years back) of a few elective courses in his specialty does not of necessity give him and his similarly fortunate fellows the copyright on all present and future knowledge in his chosen line of work. If *he* has studied and learned, others may still do so, if they have not done so already. The assumption that one who has worked in other fields for years back may never approach him or his department teachers, in either knowledge or technique, is a barrier to continuity, that can only be removed by repeated compulsory investigations.

However on the question of a *knowledge of the subject matter* the burden of blame lies chiefly on the junior school.

In methods of instruction the burden is quite apt to be shifted to the senior teacher's shoulders.

In Annapolis where our naval officers are trained, there used to be and possibly still is, the custom of appointing as instructors, officers who were specialists and experts each in his chosen field. The young middies then came to their classrooms after a night of study, prepared to prove to the instructor that they had mastered the tasks assigned them the day before. The officer-teacher was not expected to "teach" as we understand the term. Instead, he questioned, quizzed, probed and tested the *self*-education of the students before him.

We all have seen a high-school period conducted on no very different basis. The complaints that the elementary school product does not know how to study rises from such a classroom, while the elementary school answer that the high-school teacher does not know how to teach finds justification in this same room. Both charges are undoubtedly founded on facts.

However, as a result of personal visits, more and more there grows upon the high-school teacher an appreciation of the fact that the mere presence of a new pupil in his room does not justify his putting that pupil at once on the defensive to prove that he should not be marked a failure. More and more the high-school teacher becomes convinced that his duty is not to *pre-suppose* a vital interest, but rather to *create* one, if that is possible, by his own methods of daily instruction. Convince the high-school teacher through his required visits that a pupil is *able* to go on and you force him to the conclusion that to *lead* the pupil on is his bounden duty.

Following this better knowledge of the subject matter by the one and the methods by the other, comes a sympathetic understanding of each other's difficulties that makes for continuity in work such as no mere "supervision" (whether by superintendent, principal or supervisor-specialist) could ever hope to secure.

In summary, we may secure continuity in secondary work by (1) continuity of plan, which is secured by having one man or one group of men in conference prescribe the work in any

given locality, for both junior and senior schools. Then by (2) continuity of supervision, secured by having as supervisors those who had had experience as classroom teachers in both elementary and high schools. (3) By continuity of instruction, secured by compulsory frequent exchanges of visits and so of ideas by the classroom teachers in the two schools.

In advance of complete agreement in matters of administration a great deal can be accomplished by the teachers of the two types of schools, if they will get together and agree upon what is both just and reasonable in the matter of ninth year work.

In New York City twenty-seven junior high schools, or as they are called here, intermediate schools, were organized within a very short period of time, in many cases without a complete staff and in other cases without a corps of teachers prepared to conduct the courses in special branches, such as French, algebra and high school science. While this condition is temporary and the teaching positions are quickly being filled with those who have the requisite professional training, nevertheless even without specialists, the earnest and generous cooperation of the New York City high-school teachers has accomplished wonders for a better union of the two schools. Committees appointed last November made up of one half of high-school and one half of intermediate-school teachers have recently handed in reports in which there is unanimous agreement concerning the work of the ninth school year in all the major subjects. Through these agreements on the one hand the junior high-school teachers know what they are expected to furnish (and what they agree to furnish) in the line of preparation, while on the other hand the high-school teachers know what they may expect to receive and what they have agreed to accept as satisfactory.

Were it possible for any school district large or small to secure results such as these from voluntary work on the part of its teachers much that I have suggested from the standpoint of administration would be entirely unnecessary.

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NEWS AND NOTES.

The MATHEMATICS TEACHER attempts, through the News and Notes Section, to keep its readers informed concerning current activities, news items, programs of future meetings, and accounts of articles which have appeared in other publications.

Philadelphia is conducting a self survey of its schools. J. A. Foberg, recently appointed state supervisor of mathematics, is conducting the survey of mathematics.

Professor W. W. Hart of the University of Wisconsin was elected president of the Central Association of Science and Mathematics Teachers; M. J. Newell of Evanston, Illinois, was reelected chairman of the Mathematics Section.

The January number of *School Review* contains an article on "Junior High School Mathematics" by Mr. E. R. Breslich. In this paper Mr. Breslich presents an outline of the material which he recommends for the seventh grade.

On Friday, December 3, The Association of Teachers of Mathematics in New England joined with the New England Association of Colleges and Secondary Schools and several other educational associations in a dinner held at the Massachusetts Institute of Technology. The dinner was followed by brief addresses which made the occasion a fitting preliminary to the annual meeting of the Association of Teachers of Mathematics in New England which was held the following day. If the plans now under consideration are matured, there will be such a gathering of associations at the time of our annual meeting each year. The secretaries of the associations which participate in this meeting will comprise a committee to formulate a program for the two or three days devoted to the individual and joint meetings.

The annual meeting of this association was held at the Massachusetts Institute of Technology, and the following program was offered:

How we Teach Equations to Beginners, Harry C. Barber, English High School, Boston.

Magic in the Algebra Class, Henry P. McLaughlin, English High School, Boston.

How I Teach Locus Problems in Geometry, Fred D. Aldrich, Worcester Academy.

The Mathematics of Insurance, Professor Clinton H. Currier, Brown University.

Graphical Methods of Computation, Professor Joseph Lipka, Massachusetts Institute of Technology.

The following officers were elected for 1921: President, Walter F. Downey, English High School, Boston; Vice-President, Professor Clinton H. Currier, Brown University; Treasurer, Harold B. Garland, High School of Commerce, Boston; Members of Council, William L. Vosburgh, Boston Normal School, Miss Gertrude E. Preston, Dana Hall, Wellesley, Mass. (Contributed by H. D. Gaylord.)

The first meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at Hartford, Connecticut, November 6, 1920. The following program was given: Morning—Informal Reception, Business Meeting, Some Geometric Notions by Mr. Murtach M. S. Moriarty, Holyoke High School; The Need of More Fundamental Training in the Fundamental Arithmetical Processes for the Student of the Exact Sciences by Professor Emma P. Carr, Head of Department of Chemistry, Mount Holyoke College; Afternoon—Preliminary Report of the National Committee on College Entrance Requirements by Mr. Walter F, Downey, English High School, Boston; The Mathematics of the Freshman Year at Yale by Professor W. R. Longley, Sheffield Scientific School, Yale University.

Mr. Henry B. Marsh, Technical High School, Springfield, Mass., is the Chairman of the Executive Committee of the Connecticut Valley Section. (Contributed by H. D. Gaylord.)

Mr. H. E. Webb of the Central High School, Newark, N. J., has contributed a number of discussions in *The American Mathematical Monthly.* The latest paper, "Complex Numbers in Advanced Algebra," appears in the November issue.

Mr. J. Calvin Funk, head of the department of mathematics in Polytechnic High School, Mill Valley, California, writes, in the journal of the State Teachers Association:

"While the high school should articulate with the college, yet its true function is to serve not only those who desire to go to college, but also the rank and file, ninety per cent. of whom will not enter the doors of a higher institution of learning. It is absolutely unjust for the high school to take advantage of the relatively helpless situation of the student and throw at him any kind of mathematics in a form that has very little meaning to him with reference to his life career. That is what the schools are doing by teaching conventional mathematics in the first two years, and especially in the first year."

The Mathematics Section of the New York State Teachers Association met in Rochester, November 23, in connection with the annual meeting of the association. The attendance was larger than for several years past, about 300 being present.

The program was as follows:

The Proper Correlation of Intermediate Algebra, Trigonometry and Solid Geometry in the Senior High School, G. R. Merick, East High School, Rochester, N. Y.

Recent Tendencies in Secondary Mathematics, Dr. Fletcher Durell, Lawrenceville, N. J.

Suggestions on the Teaching of Mathematics from Observations in the Field, F. Eugene Seymour, Specialist in Mathematics, Education Department, Albany, N. Y.

Junior High School Mathematics, J. W. Young, Chairman of National Committee on Mathematical Requirements.

Motivation of First Lessons in Junior High School Geometry. Illustrated by lantern slides. William Betz, Rochester. (Contributed by R. L. Countryman.)

From the report of the Missouri Section of the Mathematical Association of America: "The committee appointed by the Section to co-operate with the National Committee presented through its chairman, Professor E. R. Hedrick of the University of Missouri, a report on the preliminary report on Junior High School Mathematics by the National Committee. The opinions of the Sectional Committee were, in general, favorable to the recommendations made by the National Committee, but a considerable number of detailed remarks and suggestions were made. The only one of these which is far-reaching

enough to be mentioned here is the recommendation to the National Committee that the work in demonstrative geometry ought not to be included in the junior high school. The committee was entirely in favor of leading up to demonstrative geometry, but felt that the work in the junior high schools should stop short of actual demonstrative work.

The following officers were elected for the coming year: Chairman, Professor Louis Ingold, University of Missouri; Vice-chairman, Professor Robert R. Fleet, William Jewell College; Secretary-Treasurer, Professor Paul R. Rider, Washington University." (By Professor Paul R. Rider, Secretary-Treasurer.)

The Mathematics Section of the Missouri Society of Science and Mathematics Teachers held its annual meeting November 11, in Kansas City, William A. Luby presiding. Professor U. G. Mitchell, of Kansas University, addressed the meeting on 'The Unification of Elementary Mathematics.' Mr. Austin C. Andrews speaking for the Kansas City committee appointed to study the reports of the National Committee, gave a brief review of the Preliminary Report on Junior High School Mathematics. Mr. Alfred Davis of St. Louis addressed the meeting on the work and the importance of the National Committee and urged the cooperation of every teacher of mathematics during the current school year. Mr. Davis also explained the purpose and the needs of the National Council of Teachers of Mathematics and of its journal, THE MATHEMATICS TEACHER. The section voted to become an institutional member of the Council, and a number of individuals joined.

The meeting was largely attended. Many who wished to attend could not enter the crowded room. There is a vital interest in mathematics among the teachers of Missouri.

The officers elected for the following year are: President, Alfred Davis, Soldan High School, St. Louis; Secretary, Eula A. Weeks, Cleveland High School, St. Louis.

Dr. E. R. Hedrick, of the University of Missouri, addressed the Division of Secondary Schools of the Missouri State Teachers' Association assembled in Kansas City, November 12.

His topic was 'A Revised Course in Secondary Mathematics.' The work of the National Committee on Mathematical Requirements was the theme. Dr. Hedrick said that no one should hereafter express an opinion on the content of secondary courses in mathematics who had not read the reports of this Committee. (Contributed by a member.)

Notes of the Mathematics Club of St. Louis and vicinity: At the first meeting, in November, Miss Riefling, of the Soldan High School, discussed recent publications of interest to teachers of mathematics. Charles Ammerman, of the McKinley High School, led a round table discussion on factoring in elementary algebra. Miss Eula Weeks, of the Cleveland High School, who is also a member of the National Committee on Mathematical Requirements, led a live discussion on elective courses in high-school mathematics. It is hoped that more advanced courses than are usually given in high schools might be included in these courses. However, they are to pre-suppose two years of required mathematics in the ordinary high school.

The club plans to devote most of its meetings this year to cooperating with the National Committee in the discussion of its reports as they appear. December 4, Mr. J. A. Foberg, vice-chairman of the National Committee, led the discussion on Junior High School Mathematics. January 8, Professor Hedrick discussed The Function Concept in Secondary School Mathematics. February 5, Dr. Paul Rider, of Washington University, discussed College Entrance Requirements in Mathematics. All of these discussions are to be based on reports of the committee. Later in the year the National Committee will be the guests of the St. Louis Club. The officers for the year are: President, Alfred Davis, Soldan High School; Secretary-Treasurer, Meta Eitzen, Yeatman High School. (Contributed by Mr. Davis.)

The annual meeting of the Mathematics Section of the Washington State Educational Association was held at Yakima, October 28. Professor Walter C. Eells of the department of Applied Mathematics, Whitman College, gave an address on the "Work of the National Committee on Mathematical Requirements" and discussed in detail their provi-

sional report on Junior High School Mathematics. The entire session was devoted to a spirited discussion of the report of the committee. The sentiment of the meeting on the whole was very favorable to the report.

One suggestion that was put forward with considerable vigor was that there should be a differentiation at the end of the second high-school year, and that the third year should be separated into three or four courses, one designed for girls with application to household management, etc.; one for industrial pursuits; one looking toward business, and one for Academic or College Entrance requirements.

The most serious objection to the feature of the report which suggests that the mathematics courses of the junior high school should be considered as a unit, was that there were few, if any, texts suitable for such a course.

A committee from the section was appointed to consider the report in detail and to present its findings to the National Committee at an early date. Mr. R. E. Cook, Superintendent of Schools at Chehalis, was elected chairman of the section for the coming year.

The Mathematics Section of the Nebraska State Teachers Association met November 5, 1920, with Miss Stella B. Kirker presiding. Dr. A. L. Gandy of the State University, who has made an extensive study of mathematics in European countries and the United States, gave his findings by means of the lantern and slides. It developed that in nearly all of the countries mathematics (in some form) is a required subject for a greater length of time than in the United States.

Dr. J. W. Young, of Dartmouth College, gave a summary of the mathematical requirements which have been recommended by the National Committee. During the session Mr. Young asked that a committee be appointed to represent the state in this work and to spread the reports abroad among the teachers of the state. A committee of three has been appointed with Mr. C. W. Brenke, of the State University, Lincoln, as chairman. (Contributed by Lorena J. Lewis, Secretary.)

The Mathematics Section of the Central Ohio Teachers' Association met in Columbus, Ohio, October 29, 1920. Mr. E. Forrest Bobb, Springfield High School, presided.

The first number on the program was a paper on 'What are the Essentials?—A plea for a more intelligible vocabulary in teaching Algebra,' by J. C. Boldt, Steele High School, Dayton. Mr. Boldt made a plea for the elimination of overworked terms, such as 'change of signs,' 'cancel,' and 'transposition.' He thought that multiplication should be taught before subtraction or parentheses and that nests of parentheses should be eliminated.

Mr. Branter led in the discussion of this paper. He agreed in the main with what Mr. Boldt said but saw no reason for eliminating the word 'transposition.'

The second paper on 'Some Experiments in Teaching Geometry' was presented by Mr. L. E. Coulter, Douglah Intermediate School, Columbus, Ohio. He stressed the fact that the teacher must understand both the subject matter and the child. From selections in a set of exercises he has worked out, he showed how the pupil is led to discover truth for himself and to demonstrate the propositions of geometry.

In the business session which followed, the report of the National Council of Teachers of Mathematics was endorsed.

The newly elected officers for 1921 are Mr. Edward Branter, of Springfield, president, and Miss Anna C. Mason, Columbus, secretary. (Contributed by Amy F. Preston, Secretary.)

DISCUSSION.*

Answer to Q. 4. Your question concerning the meaning of the three terms *correlated*, *fused* and *general mathematics* may perhaps be answered as follows:

These terms are still used rather vaguely and indiscriminately by authors. To me their meaning is this: The first two (correlation and fusion) refer to the manner in which the material is arranged or interwoven. The third (general mathematics) refers to the selection of material and not necessarily to its arrangement. That is, a text in "general mathematics" might follow the old compartment plan, or might employ either correlation or fusion. In the case of *correlated* mathematics each subject maintains its identity, is treated in separate chapters or in different books. But every opportunity is used to establish bonds of connections between the ideas, principles, and processes contributed by each subject. (It is the plan followed so largely in European schools where two or more mathematical subjects are taught side by side, but in close application with each other.)

Fusion implies that the separate treatment of the mathematical subjects has been given up. Fusion may be partial or complete. Thus, for many years foreign authors have tried to fuse plane and solid geometry. Again, trigonometry and geometry may be closely interwoven. Very few serious attempts have been made to blend *all* phases of elementary mathematics. The result so far seems to have been confusion. Thus, algebra and *elementary* geometry can be "correlated" or "fused" only in the field of mensuration. As soon as logical

* This department of the TEACHER is conducted by the Associate Editor, Eugene R. Smith, Headmaster of the Park School, Baltimore, Md. All correspondence should be addressed to Mr. Smith. Readers are invited to submit questions or answers in the field of their special interests. The editor in charge will refer questions to persons specially qualified to answer them. Some, or all, of the following questions will be answered in the February issue.

demonstration appears in geometry, with its chains of connected theorems, *fusion* fails.

Texts in *general* mathematics, whatever arrangement they may use, attempt to select only topics of vital importance, material which is likely to serve the "general reader." Without a far-reaching analysis of the cultural and utilitarian significance of mathematics in everyday life a text in general mathematics must remain the author's subjective interpretation of the position which mathematics occupies in the affairs of the general reader.

WM. BETZ, Rochester, N. Y.

Q. 5. If the mathematics teachers in a school are given the responsibility of selecting one of their group for the headship of their department, what qualities should they look for, and how should they be weighted? R. B. D.

Q. 6. Is it advisable to have children check the solution of an equation when the check is far more difficult than the solution? A. R. W.

NEW BOOKS.

Junior High School Mathematics, Books I., II. and III. By THEODORE LINDQUIST. Charles Scribner's Sons, New York.

Each year brings additional evidence that the mathematical curriculum of the intermediate grades is in a transition stage. The old order of straight arithmetic in the seventh and eighth grades, followed by algebra in the ninth, is giving way to a three-year course in so-called *general mathematics*. In this new course we are likely to find an earlier introduction of the simpler and more important phases of algebra, intuitive geometry and trigonometry. Arithmetic may appear in all three grades, or it may be postponed until the latter part of the course. Fewer and fewer *straight* arithmetics are coming from the press; more and more *Junior High School* series are appearing.

The outstanding characteristics of the *Lindquist* series may be summarized as follows. First, the books include materials from arithmetic, algebra, geometry, and trigonometry. Second, the social and economic material, *i.e.*, *business practice* is postponed until the ninth grade, at which time it receives a full year's treatment. This is a distinct innovation in Junior High School texts. There is much to commend the delayed treatment of such topics. Whether an entire year should be devoted to this commercial material is an open question. Third, the material in Book II. (for the eighth grade) is decidedly heavy and questionable, even for ninth-grade pupils. We refer to factoring the difference of two trinomial squares and the sum and difference of two cubes, the manipulation of fractions which contain trinomial denominators, operations with fractional exponents, square root of polynomials, radical equations, graphs of simultaneous quadratics, and logarithms. This selection of topics is clearly out of the spirit of recent developments. Any author who advocates such material even in the ninth grade will find himself at variance with the best body of opinion in the entire country, the recommendations of the National Committee on Mathematical Requirements. Fourth, the series contain an unusually attractive body of geometrical material. The illustrations are clear cut and well selected. Timely historical notes and well-motivated situations indicate that children would find the course interesting.

The Adventures of X. By MARY L. CLARK, WALLACE A. NEWLIN, AND ARTHUR E. SMOTHERS. D. C. Heath & Co., New York. Pp. 43.

This little play, in three acts, was prepared "for the purpose of adding life and interest to the teaching of algebra." It has been produced before the high school at Pasadena, and before the State High School Teachers' Association of California. Every teacher of algebra, and many pupils who have completed a year of the subject will follow the adventures of X with interest.

Fundamental Drills, Books I., II., III. and IV. By E. L. WILLIARD. The Palmer Company, New York.

These little pamphlets contain drill material in arithmetic, to be used in the elementary school. There is a need for such practice exercises, provided they are designed to exercise the pupil on the more difficult combinations. If the author selected the content of these pamphlets on basis of any careful study of the relative difficulty of the combinations, he neglected to confide it to his readers.

Rational Arithmetic. By GEORGE P. LORD. The Gregg Publishing Company, New York. Pp. 151.

This book is intended for use in business colleges and commercial high schools.

It is perhaps a bit more simplified than many of the standard texts in this field.

SECOND ANNUAL MEETING NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS.

ATLANTIC CITY, NEW JERSEY.

THURSDAY, MARCH 3, 1921.

Afternoon Session—2 to 5 P.M.

Journals of Europe for Teachers of Secondary Mathematics, Dr. David Eugene Smith, Teachers College.

Plans and Policy of THE MATHEMATICS TEACHER, J. R. Clark, Editor, The Lincoln School, New York City.

Round Table Discussion. Topics:

1. How can the National Council best serve the cause of Mathematics?
2. How can THE MATHEMATICS TEACHER be of the Greatest Service to the Teachers of Mathematics?

C. B. Walsh, Friends School, Philadelphia.
W. E. Breckenridge, Stuyvesant High School, New York.
E. R. Hedrick, University of Missouri, Columbia, Mo.
Miss Litia Odell, North Side High School, Denver, Col.
W. W. Jones, North Central High School, Spokane, Wash.

Evening Session—7 to 10 P.M.

Illustrated Lecture—Geometry and Appreciation in Nature and Art, W. E. Betz and Raleigh Schorling, The Lincoln School, New York City.

The Work of the National Committee on Mathematical Requirements, Dr. J. W. Young, Dartmouth College.

Discussion:

C. L. Thiele, Supervisor of Arithmetic, Detroit.
Wm. H. McAndrew, Associate Superintendent, New York.
L. W. Colwell, Principal Cleveland School, Chicago.
Miss Mary Faloney, Lattimer Junior High School, Pittsburgh.
E. H. Taylor, Eastern Illinois Normal, Charleston, Ill.
J. Calvin Funk, Polytechnic High School, Mill Valley, California.